

Electroweak Bosons, Leptons and Han-Nambu Quarks in a Unified Spinor-Isospinor Preon Field Model

H. Stumpf

Institut für Theoretische Physik, Universität Tübingen

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The model is defined by a selfregularizing nonlinear spinor-isospinor preon field equation and all observable (elementary and non-elementary) particles are assumed to be bound states of the quantized preon field. In a series of preceding papers this model was extensively studied. In particular for composite electroweak bosons the Yang-Mills dynamics was derived as the effective dynamics of these bosons. In this paper the first generation of composite leptons and composite Han-Nambu quarks is introduced and together with electroweak bosons, these particles are interpreted as “shell model” states of the underlying preon field. The choice of the shell model states is justified by deriving the effective fermion-boson coupling and demonstrating its equivalence with the phenomenological electroweak coupling terms of the Weinberg-Salam model. The investigation is restricted to the left-handed parts of the composite fermions. Color is revealed to be a hidden orbital angular momentum in the shell model and hypercharge follows from the effective coupling. The techniques of deriving effective interactions is a “weak mapping” procedure and the calculations are done in the “low” energy limit.

PACS 11.10 Field theory

PACS 12.10 Unified field theories and models

PACS 12.35 Composite models of particles

1. Introduction

The increasing complexity of experimental results on the lepton-quark level and of their theoretical description by means of gauge theories with high-dimensional non-abelian gauge groups etc. has led to speculations about subquark (and sublepton) models and theories. Within a short period of time a large number of papers was published in this field, cf. the review article of Lyons [1]. However, the use of such models seems only to be justified, if on the one hand they are remarkably less complicated than models on the lepton-quark level and if on the other hand such models allow a well defined quantum field theoretical formulation in order to obtain quantitative results. With respect to both these conditions the present status of subquark theories leaves much to be desired. Hence a further elaboration and discussion of subquark theories are necessary.

Concerning the first condition for the application of such models, simplicity is achieved by the compositeness hypothesis. For relativistic particles this

hypothesis was inaugurated by Jordan [2] who inferred the compositeness of the photon on the basis of a statistical argument. Subsequently de Broglie [3] put forth the proposal that the photon is composed of a neutrino and an antineutrino. This was further developed by Jordan [4], Kronig [5] and other authors. Later on, de Broglie [6] extended his proposal to a general theory of fusion of particles of spin $1/2$, and along these lines Proca [7] and Kemmer [8] derived wave equations for composite mesons, whereas Tonnelat and Petiau [9] and de Broglie [6] already discussed composite gravitons. A more formal approach to higher spin wave equations was performed by Dirac [10], Fierz [11], Pauli and Fierz [12], Bargmann and Wigner [13] and other authors. The calculus of fusion was, however, not only applied to bosons but also used for speculations about composite fermions, formally described for instance for three fermions by equations of Rarita and Schwinger [14]. Guided by experimental results Hönl and Boerner [15] speculated that the proton belongs to a degree of fusion $N > 3$, i.e. has to be composed of more than three elementary fermions. With the further development of relativistic quantum field theory it became obvious that fusion need not be strictly local but can be a non-local phenomenon. Bopp [16] generalized de Broglie's

Reprint requests to Prof. Dr. H. Stumpf, Institut f. Theoretische Physik der Universität Tübingen, Auf der Morgenstelle 14, D-7400 Tübingen.

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ansatz to many-particle equations with relativistic potentials and proposed leptons to be composed of three neutrinos. Finally, in the nonlinear spinor field approach of Heisenberg [17] any elementary particle is assumed to be a bound state of elementary spinor-isospinor fields.

In spite of its simplicity the radical assumption of de Broglie and Heisenberg was not generally accepted. Due to the development and the great success of gauge theories and the introduction of supersymmetry it is a widespread belief that for instance gauge bosons are elementary objects. Nevertheless, in the course of the development of high energy physics an increasing number of bosons and gauge bosons which were initially assumed to be elementary turned out to be composite objects. For instance, in quantum chromodynamics mesons are quark-antiquark composites etc. and gluons are elementary. However, on the preon level gauge gluons are preon composites and also the electroweak gauge bosons are assumed to be composites. Thus the problem of compositeness leads to the problem whether there is at all a final microscopic quantum field theory with elementary objects and whether these objects are only fermions or fermions and bosons. While the first question will probably never find a final answer, with respect to the second question we assume the de Broglie-Heisenberg hypothesis to be valid on the preon level. This means: For the theoretical formulation of a suitable preon model we use as a basic quantity a spinor-isospinor field obtaining in this way a maximal simplicity. Furthermore, due to the de Broglie-Heisenberg hypothesis the basic field dynamics may only contain spinor-isospinor preon field interactions, i.e. it leads to a nonlinear spinor field model.

For the formulation of his spinor field model on the nucleon level Heisenberg [17] used a nonlinear spinor field (NSF) equation with first order derivatives (FDNSF equation) and local interactions. Such a theory is non-renormalizable and the attempts of Heisenberg to use an ad hoc dipole ghost regularization failed. Thus in order to avoid this drawback we have to reformulate the basic spinor field equation. In doing so we fulfill our second condition in order to obtain a well defined quantum field theory for our model.

To circumvent the non-renormalizability of FDNSF equations higher order derivative nonlinear

spinor field (HDNSF) equations can be used. Such equations exhibit selfregularizing properties as was first discovered by Bopp [18] and Podolsky [19] for the case of scalar wave equations with higher order derivatives. Afterwards many authors worked in this field but no general results were obtained because these models lead to indefinite metric which need to be separately discussed for each model. Thus in our HDNSF-model the treatment of composite particle theory is partially merged with the solution of problems related to the special model under consideration. This is the prize one has to pay for the extreme simplicity of HDNSF models connected with the assumption that only elementary fermions are the constituents of matter.

The assumption of elementary (unobservable) preon fields includes that all observable (elementary and non-elementary) particles are bound states of these fields. Therefore, the existence and the processes of observable particles are governed by the formation and reactions of relativistic composite particles, i.e. of multipreon states, while elementary preons are confined. Thus a relativistic composite particle quantum (field) theory is needed to describe the physics of observable particles. Although numerous efforts were made in the past to develop such a relativistic composite particle theory, no satisfactory and systematic answers have so far been obtained in the literature for the solution of this problem. Therefore, a research program has been started by the author and collaborators in this field.

The preceding investigations about the HDNSF preon model are contained in papers of Grosser and Lauxmann [20], Grosser [21], Grosser, Hailer, Hornung, Lauxmann and Stumpf [22] and of the author [23–25]. An extensive discussion of the results and of the mathematical techniques is given in [24]. The general basis of this approach is the so-called functional quantum theory, a formulation of quantum field theory which prefers to work mainly with states and not with operators as was elaborated by the author [26]. Such a formulation is forced by the indefinite metric problem. After having formulated the model and obtained a first insight into the formation of bound states etc. in [20–23] the most urgent problem is the investigation of effective interactions between bound states, i.e. relativistic composite particles. In particular it has to be proved that relativistic composite particles representing observable “elementary” particles satisfy in certain

approximations the corresponding gauge theories etc. which govern the reactions of these particles, if they are considered to be elementary and pointlike. A first step in this direction was performed by the author [24] where it was demonstrated that the effective interactions of composite fermions and composite scalar bosons in an HDNSF model lead to a Yukawa theory in the low energy range while in the high energy range formfactor corrections appear. The mathematical techniques used for these investigations was a “weak mapping” of the HDNSF model on to a Yukawa theory. While the composite particle states in these papers have to be considered only as “test” states without immediate physical meaning, in a subsequent paper of the author [25] the weak mapping of a special HDNSF model on to a Yang-Mills theory, i.e. the effective dynamics of composite massless vector bosons for the case of a non-abelian local $SU(2)$ gauge group was studied. As a result it turned out that the local gauge group is exclusively generated by the bound state many particle dynamics itself and that it need not be incorporated a priori in the HDNSF model, i.e. the HDNSF model contains only the global $SU(2)$ (isospin) group. In this paper this investigation is continued by the calculation of the coupling between such non-abelian (electroweak) $SU(2)$ and $U(1)$ gauge bosons and corresponding fermions, leading to definite conclusions about the wave functions of leptons and Han-Nambu quarks, and their electroweak coupling constants to the gauge bosons. For the sake of brevity we will not discuss in this introduction the various steps of the calculations which are performed in the following. Rather we restrict ourselves to some comments about the relations to the work of other authors.

Usually in quantum field theory the effective dynamics of composite particles (as a matter of fact, however, of composite fields) is studied by operator mappings (strong mappings). The history of strong mappings, its drawbacks and its comparison with weak mapping was extensively discussed by the author in [23–25]. So we refer with respect to our preference of weak mapping to these papers and need not again discuss this topic here.

With respect to the isospin degrees of freedom Harari [27] and Shupe [28] proposed a model with similar simplicity. In this model leptons, quarks, gauge bosons etc. are constructed from two basic entities (rishons, squibs), one with fractional charge

$1/3$ and the other with zero charge. In this model some important facts in the lepton-quark world find a very simple explanation. However, the first investigations were done only in a combinatorial way without the use of any quantum theoretical calculation scheme. By improving such considerations within the framework of a quantum theory it soon turned out that difficulties appeared. For instance, the original proposal of Harari and Shupe for the explanation of color could not be maintained, cf. Sogami [29]. Thus in constructing a quantum field theoretical dynamics for this rishon-squib model Harari and Seiberg [30] were led to the introduction of color and hypercolor forces mediated by corresponding gauge bosons. Such assumptions circumvented the above mentioned difficulties. On the other hand, the model now contained at least 34 elementary particles and simplicity is lost. Developing the Harari-Shupe model on a purely combinatorial level Elbaz [31] achieved interesting results but was forced to assume additional color degrees of freedom from the beginning, so even at this level the situation with respect to simplicity is unsatisfactory. In contrast to this isospin model with fractional charges, our model contains only integer charges, i.e. one isospin state (P) with unit charge, and the other isospin state (N) with zero charge and additionally we will see that color need not be introduced a priori but can be easily derived as a hidden orbital angular momentum.

Finally, Dürr and Saller continued the Heisenberg approach and published numerous papers about attempts to deduce all other gauge groups from a basic global $SU(2)$ spinor field group, cf. for instance Dürr [32], Saller [33], Dürr and Saller [34]. These authors use operator techniques and concentrate on symmetry breaking etc. Their approach is complementary to the approach followed here. So it might be of interest to study such different points of view but it is not necessary to enter into a detailed discussion and comparison.

2. Fundamentals of the model

The general unified nonlinear preon field model which is assumed to be the basis of the theory is defined by the field equations

$$\begin{aligned} & [(-i\gamma^\mu \partial_\mu + m_1) (-i\gamma^0 \partial_0 + m_2)]_{\alpha\beta} \psi_\beta(x) \\ & = g V_{\alpha\beta\gamma\delta} \psi_\beta(x) \bar{\psi}_\gamma(x) \psi_\delta(x), \end{aligned} \quad (1.1)$$

where the index α is a superindex describing spin and isospin. Due to the mass terms in (1.1) the corresponding spinor field has to be a Dirac-spinor-isospinor.

In contrast to the non-renormalizability of FDNSF models and the difficulties connected with this property the model (1.1) exhibits self-regularization, relativistic invariance and locality for common canonical quantization.

For the further evaluation equation (1.1) has to be decomposed into an equivalent set of FDNSF equations. It was proposed by the author [23] and Grosser [21] that the set of nonlinear equations $r = 1, 2$

$$(-i\gamma^\mu \partial_\mu + m_r)_{\alpha\beta} \varphi_{\beta r}(x) = g \lambda_r \sum_{stu} V_{\alpha\beta\gamma\delta} \varphi_{\beta s}(x) \bar{\varphi}_{\gamma t}(x) \varphi_{su}(x) \quad (1.2)$$

is connected with (1.1) by a biunique map where this map is defined by the compatible relations

$$\begin{aligned} \psi_\alpha(x) &= \varphi_{\alpha 1}(x) + \varphi_{\alpha 2}(x), \\ \varphi_{\alpha 1}(x) &= \lambda_1 (-i\gamma^\mu \partial_\mu + m_1)_{\alpha\beta} \psi_\beta(x), \\ \varphi_{\alpha 2}(x) &= \lambda_2 (-i\gamma^\mu \partial_\mu + m_2)_{\alpha\beta} \psi_\beta(x) \end{aligned} \quad (1.3)$$

with $\lambda_r := (-1)^r (2\Delta m)^{-1}$ and $\Delta m := 1/2(m_1 - m_2)$.

The quantization of the model was performed in [22] and [23]. It turns out that the $\psi, \bar{\psi}$ anticommutator vanishes for equal times, and therefore in the interaction term of the corresponding Lagrangian all field operators completely anticommute. This is an essential property required for an unambiguous consistent definition of the vertex operator. Due to the condition of forminvariance of the corresponding Lagrangian density against Poincaré-transformations, global isospin transformations and permutations of the field operators, the vertex operator must have the general form

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2} \sum_h (v_{\alpha\beta}^h v_{\gamma\delta}^h - v_{\alpha\delta}^h v_{\gamma\beta}^h). \quad (1.4)$$

If we explicitly introduce the spinor-isospinor indexing by writing $\alpha \equiv (\alpha, A)$ and assume the unit operator to be the vertex in isospin-space then v^h reads

$$v_{\alpha\beta}^h \equiv \hat{v}_{\alpha\beta}^h \delta_{AB}. \quad (1.5)$$

With respect to the remaining spinorial vertex part we assume the most general chiral invariant scalar and pseudoscalar coupling $h = 1, 2$ with

$$\hat{v}_{\alpha\beta}^1 := \delta_{\alpha\beta}; \quad \hat{v}_{\alpha\beta}^2 := i\gamma_{\alpha\beta}^5 \quad (1.6)$$

which was used in the symmetry breaking FDNSF-model of Nambu and Jona-Lasinio [35] and which is equivalent to the vector and pseudovector coupling of Heisenberg's FDNSF equation [36] due to the Fierz theorem. Although the kinetic part of (1.2) is not chiral invariant, this kind of coupling is needed to obtain mass-zero bound boson states in the theory as was demonstrated in [25].

For the following investigation it is convenient to replace the adjoint spinors by the charge conjugated spinors as was done in [25]. The charge conjugated spinor is defined by

$$\varphi_{Ajv}^c = C_{vv}^{-1} \bar{\varphi}_{Ajv'} \quad (1.7)$$

and introducing the superspinors

$$\begin{aligned} \varphi_{Aj\alpha 1} &:= \varphi_{Aj\alpha}, \\ \varphi_{Aj\alpha 2} &:= \varphi_{Aj\alpha}^c \end{aligned} \quad (1.8)$$

we can combine (1.2) and its charge conjugated equation into one equation [25]

$$\begin{aligned} &\sum_{Z_2} (D_{Z_1 Z_2}^\mu \partial_\mu - m_{Z_1 Z_2}) \varphi_{Z_2} \\ &= \sum_{h Z_2 Z_3 Z_4} U_{Z_1 Z_2 Z_3 Z_4}^h \varphi_{Z_2} \varphi_{Z_3} \varphi_{Z_4} \end{aligned} \quad (1.9)$$

with $Z := (\alpha, A, i, A)$ and

$$\begin{aligned} \alpha &= \text{spinor index } (\alpha = 1, 2, 3, 4), \\ A &= \text{isospinor index } (A = 1, 2), \\ i &= \text{auxiliary field index } (i = 1, 2), \\ A &= \text{superspinor index } (A = 1, 2), \end{aligned} \quad (1.10)$$

where the following definitions are used

$$\begin{aligned} D_{Z_1 Z_2}^\mu &:= i\gamma_{\alpha_1 \alpha_2}^\mu \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ m_{Z_1 Z_2} &:= m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ U_{Z_1 Z_2 Z_3 Z_4}^h &:= g \lambda_{i_1} \hat{v}_{\alpha_1 \alpha_2}^h \delta_{A_1 A_2} \delta_{A_1 A_2} (\hat{v}^h C)_{\alpha_3 \alpha_4} \delta_{A_3 A_4} \delta_{A_3 1} \delta_{A_4 2}. \end{aligned} \quad (1.11)$$

The quantum states of the model (1.1) or (1.9) respectively are described by state functionals $|\mathfrak{T}[j, a]\rangle$ with respect to the states $\{|a\rangle\}$ where $j \equiv j_Z(x)$ are sources with corresponding Z -indices. For concrete calculations it is necessary to introduce normal transforms by $|\mathfrak{T}\rangle = Z_0[j] |\mathfrak{F}\rangle$ and the energy representation of the spinor field in terms of state functionals. Both procedures were discussed in detail in [22] and need not be repeated here. The

resulting functional equation reads

$$\begin{aligned} p_0 |\mathfrak{F}\rangle = & \sum_{Z_1 Z_2} \int j_Z(x) i D_{ZZ_1}^0 [D_{Z_1 Z_2}^k \partial_k - m_{Z_1 Z_2}] \\ & \cdot \partial_{Z_2}(x) d^4 x |\mathfrak{F}\rangle \\ & + \sum_{h Z Z_1 Z_2 Z_3 Z_4} \int j_Z(x) i D_{ZZ_1}^0 U_{Z_1 Z_2 Z_3 Z_4}^h \\ & \cdot d_{Z_4}(x) d_{Z_3}(x) d_{Z_2}(x) d^4 x |\mathfrak{F}\rangle \quad (1.12) \end{aligned}$$

with

$$d_Z(x) := \partial_Z(x) - \sum_{Z'} \int F_{ZZ'}(x, x') j_{Z'}(x') d^4 x'. \quad (1.13)$$

If (1.12) is projected into configuration space it allows the transition to a one-time description of the states $|\mathfrak{F}\rangle$. Formally this can be achieved by substituting $j_Z(x) \rightarrow j_Z(\mathbf{r}) \delta(t)$ into (1.12, 1.13) and $|\mathfrak{F}\rangle$ and by writing $\partial(\mathbf{r}) \equiv \partial(\mathbf{r}, 0)$. Then we can replace (1.12) by the following equation

$$\begin{aligned} p_0 |\mathfrak{F}\rangle = & \sum_{Z_1 Z_2} \int j_Z(\mathbf{r}) i D_{ZZ_1}^0 [D_{Z_1 Z_2}^k \partial_k - m_{Z_1 Z_2}] \\ & \cdot \partial_{Z_2}(\mathbf{r}) d^3 r |\mathfrak{F}\rangle \\ & + \sum_{h Z Z_1 Z_2 Z_3 Z_4} \int j_Z(\mathbf{r}) i D_{ZZ_1}^0 U_{Z_1 Z_2 Z_3 Z_4}^h \\ & \cdot d_{Z_4}(\mathbf{r}) d_{Z_3}(\mathbf{r}) d_{Z_2}(\mathbf{r}) d^3 r |\mathfrak{F}\rangle, \quad (1.14) \end{aligned}$$

and with the abbreviations

$$\begin{aligned} K_{I_1 I_2} &:= K_{Z_1 Z_2}(\mathbf{r}_1 \mathbf{r}_2) \\ &:= i \sum_Z D_{ZZ_1}^0 (D_{Z_1 Z_2}^k \partial_k - m_{Z_1 Z_2}) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (1.15) \end{aligned}$$

$$\begin{aligned} W_{I_1 I_2 I_3 I_4}^h &= W_{Z_1 Z_2 Z_3 Z_4}^h(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \\ &:= i \sum_Z D_{ZZ_1}^0 U_{Z_1 Z_2 Z_3 Z_4}^h \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_1 - \mathbf{r}_4) \end{aligned} \quad (1.16)$$

equation (1.14) can be written in the compact form

$$\begin{aligned} p_0 |\mathfrak{F}\rangle = & \sum_{I_1 I_2} j_{I_1} K_{I_1 I_2} \partial_{I_2} |\mathfrak{F}\rangle \\ & + \sum_{h I_1 I_2 I_3 I_4} j_{I_1} W_{I_1 I_2 I_3 I_4}^h d_{I_4} d_{I_3} d_{I_2} |\mathfrak{F}\rangle =: \mathcal{H} |\mathfrak{F}\rangle. \end{aligned} \quad (1.17)$$

We assume the spinorfield interaction term to be normalordered. Then by evaluating this term in the corresponding functional equation (1.17) the local terms $F_{I_2 I_3} \partial_{I_4}$, $F_{I_3 I_4} \partial_{I_2}$, $F_{I_4 I_2} \partial_{I_3}$ (in connection with the definition of W) drop out. The special form of the evaluated interaction term of (1.17) was given in [25] and need not be repeated here.

2. Leading Term Approximation

The properties and the physical meaning of composite preon states which are assumed to represent “elementary” particles can be studied by the investigation of effective interactions. In [24, 25] it was demonstrated that the appropriate method for such an investigation is the weak mapping procedure. This procedure is defined by a transformation of the set of functional preon source operators $\{j_I\}$ into a set of cluster source operators $\{X_R\}$. For the case of a mixed spectrum of two-preon boson states and three-preon fermion states the corresponding cluster source operators read

$$b_K = \sum_{I_1 I_2} C_K^{I_1 I_2} j_{I_1} j_{I_2} \quad (2.1)$$

and

$$f_R = \sum_{I_1 I_2 I_3} C_R^{I_1 I_2 I_3} j_{I_1} j_{I_2} j_{I_3}, \quad (2.2)$$

where the set of coefficient functions $\{C_K, K = 1, \dots\}$ and $\{C_R, R = 1, \dots\}$ are defined by complete sets of cluster states, i.e. such sets in general contain bound state clusters as well as scattering state clusters. The transformation of the preon source operators $\{j_I\}$ into boson and fermion source operators $\{b_K, f_R\}$ induces a transformation of the set of functional states $\{|\mathfrak{F}\rangle\}$ as well as of the corresponding functional equation (1.17). These transformations are characterized by the invariance conditions

$$|\mathfrak{F}[j, a]\rangle = |\hat{\mathfrak{F}}[b, f, a]\rangle \quad (2.3)$$

and

$$\mathcal{H} \left[j, \frac{\delta}{\delta j} \right] = \hat{\mathcal{H}} \left[b, \frac{\delta}{\delta b}, f, \frac{\delta}{\delta f} \right], \quad (2.4)$$

where $|\hat{\mathfrak{F}}\rangle$ and $\hat{\mathcal{H}}$ are the cluster transforms of $|\mathfrak{F}\rangle$ and \mathcal{H} .

The weak mapping of a mixed spectrum of two-preon scalar boson states and of three-preon fermion states was discussed in [24]. By a detailed investigation it turned out that the cluster state representation $\hat{\mathcal{H}}$ of the functional energy operator \mathcal{H} leads to a hierarchy of interactions where only the leading terms are of interest since the higher order terms have only a negligible influence on the physical reactions. A closer inspection of this result reveals that this hierarchy of interactions is generated by the normalization properties, the energetic proper-

ties and the number of participating wave functions in the various interaction terms. This situation is qualitatively not changed if we additionally take into account vector boson states and specify the fermion states in more detail by using lepton and quark states. Therefore, for the following investigation we take over the results of [24] and discuss only those terms which were proved to be the leading terms of the cluster transformation of (2.4).

Using the same notations as in [24] the general boson-fermion transform of \mathcal{H} without any approximation can be written according to [24] in the form

$$\hat{\mathcal{H}} = \sum_{k=0}^5 \mathcal{H}_{bb}^k + \sum_{k=0}^1 \mathcal{H}_{ff}^k + \sum_{k=1}^5 \mathcal{H}_{bf}^k + \sum_{k=1}^3 \mathcal{H}_{bff}^k, \quad (2.5)$$

where in comparison with [24] terms with fermion sources f and adjoint fermion sources \bar{f} were combined in a whole due to our superspinor notation.

Before discussing the various terms of (2.5) we apply to (2.5) the leading term approximation. According to [24] this approximation can be characterized by two steps:

- i) The complete sets of cluster states, i.e. of scattering states as well as of bound states are reduced to the subsets of bound states;
- ii) the complete set of transformed functional energy operator terms is reduced to the subset of highest magnitude terms.

While the first step is justified by energetic estimates (decoupling theorem, extremely heavy scattering state masses due to extremely heavy preon masses), the second step is justified by interaction estimates (perturbation theory, extremely small coupling constants). Furthermore, it is convenient to assume that only the physical relevant bound states occur and that possibly exotic bound states can be excluded by more detailed bound state investigations.

The calculations which were done under these conditions in [24, 25] yield for (2.5) the following approximate expression

$$\hat{\mathcal{H}} \approx (\mathcal{H}_{bb}^0) + (\mathcal{H}_{bb}^3) + (\mathcal{H}_{ff}^0) + (\mathcal{H}_{bf}^1) + (\mathcal{H}_{bff}^1), \quad (2.6)$$

where the brackets symbolize step i) while all other terms are omitted due to step ii). The physical meaning of the remaining terms in (2.6) is the following: The terms (\mathcal{H}_{bb}^0) and (\mathcal{H}_{ff}^0) describe the free motion of the bound boson states and of bound fermion states respectively. The terms of (\mathcal{H}_{bb}^3)

vanish for scalar bosons, but lead to the non-abelian selfinteraction for SU(2) vector boson states. The terms of (\mathcal{H}_{bf}^1) describe the coupling term of the fermion-boson interaction in the fermion field equation, while the terms of (\mathcal{H}_{ff}^1) yield the coupling term of the boson-fermion interaction in the boson field equations. In a preliminary way all terms of (2.6) were already discussed in the low energy limit as well as in the high-energy limit in [24]. A more profound discussion of the terms (\mathcal{H}_{bb}^0) , (\mathcal{H}_{bb}^3) were performed in [25] in the low energy limit exactly taking into account all algebraic complications with respect to the spinor- and the superspinor-isospinor algebra and antisymmetry permutations. This discussion led to the derivation of a Yang-Mills nonlinear field as an effective law of motion for composite vector bosons.

In this paper we take over the vector- and scalar boson states of [25] and study the interaction terms of (\mathcal{H}_{bf}^1) in order to obtain an information and justification of lepton states and quark states. If the results and the techniques of [24, 25] are used for the evaluation of these terms, we obtain the explicit expression

$$\begin{aligned} (\mathcal{H}_{bf}^1) = & 3 \sum_{hKRS} \hat{W}_{I_1 I_2 I_3 I_4}^h C_K^{I_2 I_3} C_R^{I_4 AB} R_{AB}^S f_S \frac{\delta}{\delta f_R} \frac{\delta}{\delta b_K} \\ & - 6 \sum_{hKRS} \hat{W}_{I_1 I_2 I_3 I_4}^h C_R^{I_2 I_3 A} C_K^{I_4 B} R_{AB}^S f_S \frac{\delta}{\delta f_R} \frac{\delta}{\delta b_K}, \quad (2.7) \end{aligned}$$

where the symbol R denotes the dual states with respect to the sets $\{C_K\}$ $\{C_S\}$, see [24], while \hat{W}^h is defined by

$$\hat{W}_{I_1 L_2 L_3}^h := \{W_{I_1 L_2 L_3}^h\}_{as(L_1 L_2 L_3)} \quad (2.8)$$

i.e. the vertex is antisymmetrized in the last three superindices, cf. [25].

3. Low Energy Composite Particle States

The weak mapping of the self-interacting preon field on to composite boson fields and composite fermion fields is defined by the functional relations (2.1), (2.2), (2.3), and (2.4). For an evaluation of this map the general explicit form of the corresponding wave functions of (2.1) and (2.2) is needed. According to the leading term approximation only the bound states have to be taken into account and thus only these states have to be discussed. Furthermore,

for the purposes of a first investigation of such a map it suffices to consider its low energy limit. In this case it is easier to derive the general explicit form of the wave functions under consideration than in the general case and, in addition, the low energy limit is just that range where we can compare the effective interactions of the composite particles with current phenomenological field theories, i.e. where we can scrutinize the physical relevance of our model. In order to perform this we have to abandon the formal notation of (2.1) and (2.2) and to replace it by a full indexing.

The general index I of (2.1) and (2.2) is defined by $I := (\mathbf{r}, Z) \equiv (\mathbf{r}, \alpha, A, i, A)$. For the following investigation it is convenient to combine the isospin index A and the superspin index A into a single index \varkappa by means of the map $(A, A) \rightarrow \varkappa$, i.e. by $\{(A, A) = (1, 1), (1, 2), (2, 1), (2, 2)\} \rightarrow \{\varkappa = 1, 2, 3, 4\}$. With this notation we have $I := (\mathbf{r}, i, \alpha, \varkappa)$ and (2.1) and (2.2) read with $d^3r \equiv dr$ etc.

$$b \begin{pmatrix} n \\ \mathbf{k} \\ d \\ e \end{pmatrix} = \sum_{\substack{r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa'}} \int C \begin{pmatrix} \mathbf{r}, \mathbf{r}' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \begin{pmatrix} n \\ \mathbf{k} \\ d \\ e \end{pmatrix} j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} dr dr' \quad (3.1)$$

and

$$f \begin{pmatrix} m \\ \mathbf{q} \\ l \\ j \end{pmatrix} = \sum_{\substack{r, r', r'' \\ \alpha, \alpha', \alpha'' \\ \varkappa, \varkappa', \varkappa''}} \int C \begin{pmatrix} \mathbf{r}, \mathbf{r}', \mathbf{r}'' \\ \alpha, \alpha', \alpha'' \\ \varkappa, \varkappa', \varkappa'' \end{pmatrix} \begin{pmatrix} m \\ \mathbf{q} \\ l \\ j \end{pmatrix} j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} j \begin{pmatrix} \mathbf{r}'' \\ \alpha'' \\ \varkappa'' \end{pmatrix} dr dr' dr'' \quad (3.2)$$

where we resolved the general index K and R respectively into the detailed form $K := (n, \mathbf{k}, d, e)$ and $R := (m, \mathbf{q}, l, j)$ with

n, m = internal bound state excitation index,
 \mathbf{k}, \mathbf{q} = center of mass momentum vector,
 d, l = spinor state index,
 e, j = isospin-superspinor state index.

According to [23, 24] in the low energy limit the dependence of the boson and fermion wave functions on the auxiliary field indices (r, r') and (r, r', r'') can be decoupled from the remaining vari-

ables. Taking over this result and decomposing the wave functions into an internal part and the center mass part, (3.1) and (3.2) go over into

$$b \begin{pmatrix} n \\ \mathbf{k} \\ d \\ e \end{pmatrix} = \sum_{\substack{r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa'}} U^{rr'} \int e^{i\mathbf{k}(\mathbf{r}+\mathbf{r}')/2} \chi_n^{de}(\mathbf{r}-\mathbf{r}' | \mathbf{k}) j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} dr dr' \quad (3.3)$$

and

$$f \begin{pmatrix} m \\ \mathbf{q} \\ l \\ j \end{pmatrix} = \sum_{\substack{r, r', r'' \\ \alpha, \alpha', \alpha'' \\ \varkappa, \varkappa', \varkappa''}} U^{rr'r''} \int e^{i\mathbf{q}(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')/3} \chi_m^{lj}(\mathbf{r}'-\mathbf{r}'', \mathbf{r}-\frac{1}{2}(\mathbf{r}'+\mathbf{r}'')) j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} j \begin{pmatrix} \mathbf{r}'' \\ \alpha'' \\ \varkappa'' \end{pmatrix} dr dr' dr'' \quad (3.4)$$

where the explicit form of the matrices $U^{rr'}$ and $U^{rr'r''}$ is given in [23, 24] and will not be repeated here.

As $U^{rr'}$ and $U^{rr'r''}$ respectively and the center of mass parts of the boson wave functions and fermion wave functions are symmetric with respect to an interchange of their variables, the antisymmetry condition for the complete wave function $C_K^{ll'}$ and $C_R^{ll'l''}$ can only be fulfilled by the internal part of the wave functions of (3.3) and (3.4) respectively. This condition can easily be incorporated into the internal part wave function of (3.3) but is more complicated for (3.4). Thus for further evaluation both kinds of wave functions are to be treated separately. With respect to the boson wave functions a complete structural analysis was performed in [25] for vector boson states. We first discuss these states.

Without loss of generality the internal boson wave function χ of (3.3) can be expanded in terms of a Dirac algebra with respect to (α, α') as well as with respect to (\varkappa, \varkappa') . In general linear combinations of these elements are required for the construction of true eigenstates. It was, however, demonstrated in [25] that the selfenergy part (\mathcal{H}_{bb}^0) of the single composite boson states in (2.6) is diagonalized in the low energy limit even if single elements of this

Dirac algebra are used, i.e.

$$\chi_n^{de}(\mathbf{r} - \mathbf{r}' | \mathbf{k}) = \chi_n(\mathbf{r} - \mathbf{r}' | \mathbf{k}) S_{xx'}^d T_{xx'}^e \quad (3.5)$$

are eigenstates of the “free” boson Hamiltonian (\mathcal{H}_{bb}^0) which of course contains the internal preon interactions of the single composite boson state, in contrast to the phenomenological free particle Hamiltonian. It was furthermore demonstrated in [25] that in spite of these internal interactions the resulting effective Hamiltonian of free composite bosons is identical with its phenomenological counterpart.

In the low energy limit, i.e. for small values of \mathbf{k} the dependence of the internal wave function χ in (3.5) on \mathbf{k} can be neglected as was outlined in [24]. Furthermore, it was shown that χ must be symmetric. If we assume that besides the composite boson ground state no low energy excited bound boson state exists, then we have to take into account only $n=0$, which is according to [24] a strongly concentrated spherical symmetric state in coordinate space. The antisymmetry of the total wave function can then be carried only by the algebraic parts of (3.5). In [25] it was shown that in order to obtain vector bosons S^d must be symmetric, while T^e must be antisymmetric. So the boson wave functions are completely fixed. The symmetric representation of the spin states S^d is given by

$$\{S^d\}_{\text{sym}} := \{\gamma_\mu C, \Sigma_{\mu\nu} C\} \quad (3.6)$$

while according to [25] the antisymmetric representation of the isospinor-superspinor states T^e is given by

$$\{T^e\}_{\text{antisym}} := \left\{ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \sigma_t \\ (-1)^t \sigma_t & 0 \end{pmatrix}, t = 1, 2, 3 \right\}, \quad (3.7)$$

where the first part represents the U(1) vector bosons while the second part represents the triplet of SU(2) vector bosons. Finally, we perform a further approximation of the orbital part and assume this part in the framework of a shell model description to be given by a strongly concentrated 1s function for $n=0$

$$\chi_0(\mathbf{z} | \mathbf{k}) = 2m^{3/2} e^{-mz}, \quad (3.8)$$

where $m = m_1 + m_2$. With these results for the wave functions we were able to show in [25] that the effective dynamics of these bosons is a U(1)–SU(2) Yang-Mills dynamics, a calculation which is independent of the shell model approximation (3.8).

Apart from the general considerations in [24] with respect to composite fermions no detailed calculation of the internal wave functions χ appearing in (3.4) has been made so far. Thus in accordance with the shell model idea we postulate shell model wave functions for composite fermions and justify their use by the investigation of the effective interactions of these composite fermions with bosons. Later on, χ such shell model states have to be derived by appropriate approximations of the corresponding bound state equations. Like in the two-preon case we also consider in the three-preon case only the low energy limit. In this limit it follows from the general considerations of [24] that the internal wave function χ of the three-preon states can be separated into orbital parts and algebraic parts thus leading to the general ansatz

$$\begin{aligned} \chi_m^{lj}(\mathbf{r}' - \mathbf{r}'', \mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'')) \\ = \Theta_{xx'x''}^j \chi_{xx'x''}^l \chi_m(\mathbf{r}' - \mathbf{r}'', \mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'')) . \end{aligned} \quad (3.9)$$

The spin part χ^l stems from an expansion of χ_m^{lj} in terms of Dirac spinors for the free preon fields, and it is a peculiarity of the three-preon bound state equation that in this expansion either $u \otimes u \otimes u$ terms or $v \otimes v \otimes v$ terms, but no mixed terms of u - and v -states are admitted. This means that in the $u \otimes u \otimes u$ -expansion as well as in the $v \otimes v \otimes v$ -expansion only two spin states for any particle are available. By a direct calculation of the effective interactions it will turn out that, in contrast to the boson states, the isospinor-superspinor part Θ^j for composite fermions must be symmetric. Thus in order to maintain antisymmetry for the total wave function the spin-orbital part must carry the antisymmetry. As furthermore only two spin states for any particle are admitted, in the low energy limit where $u_x(l) = \delta_{lx}$, $l=1, 2$ and $v_x(l) = \delta_{lx}$, $l=3, 4$, a totally antisymmetric spin state vanishes. Hence we are forced to use an antisymmetric wave function with respect to spin as well as to orbital parts. Taking into account the spin state expansion and assuming a product representations of the orbital parts in the spirit of the shell model we thus obtain

$$\begin{aligned} \chi_{xx'x''}^l \chi_m(\mathbf{r}' - \mathbf{r}'', \mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'')) \\ = \left\{ \begin{pmatrix} l_1 \\ \alpha \end{pmatrix} \begin{pmatrix} l_2 \\ \alpha' \end{pmatrix} \begin{pmatrix} l_3 \\ \alpha'' \end{pmatrix} g(\mathbf{r}' - \mathbf{r}'') h_e(\mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'')) \right\}_{\text{as}} \end{aligned} \quad (3.10)$$

with

$$\begin{pmatrix} l_1 \\ \alpha \end{pmatrix} \begin{pmatrix} l_2 \\ \alpha' \end{pmatrix} \begin{pmatrix} l_3 \\ \alpha'' \end{pmatrix} = \begin{cases} u_\alpha(l_1) u_{\alpha'}(l_2) u_{\alpha''}(l_3), \\ v_\alpha(l_1) v_{\alpha'}(l_2) v_{\alpha''}(l_3). \end{cases} \quad (3.11)$$

Due to the restriction of the number of spin states the functions g and h_q are not allowed to be equal since for $g \equiv h_q$ and very concentrated states in the low energy limit the norm of (3.10) would vanish. Hence we begin to fill up the next shell, i.e. we define g to be the first shell state already used in (3.8)

$$g(\mathbf{z}) := 2m^{3/2} e^{-mz} \quad (3.12)$$

while the next, the second shell is defined by

$$h_q(\mathbf{z}) = 2m^{3/2} e^{-mz} (1 - mz) \quad (3.13)$$

for $q = 0$, and

$$h_q(\mathbf{z}) = 2m^{3/2} e^{-mz} z_q \quad (3.14)$$

for $q = 1, 2, 3$. This means we have for bosons 1s-states, while fermions are described by (1s, 2s) or (1s, 2p)-states respectively in the orbital parts. It will be shown later that the 2p-states can be identified with color degrees of freedom, i.e. the phenomenological color space is a hidden orbital angular momentum space. Taking for granted this interpretation, we finally have to define the isospinor-superspinor algebra. Like for the spin part we represent the isospinor-superspinor part by direct products of single particle states. In contrast, however, to the spin expansion, the isospinor-superspinor expansion is not restricted to submanifolds. In addition, we are not forced to use for this expansion Dirac-spinors as in the isospinor-superspinor space no single preon Hamiltonian etc. have to be diagonalized. Thus it suffices to use unit vectors in this space which are defined by

$$\begin{pmatrix} l \\ \alpha \end{pmatrix} := \delta_{l\alpha}; \quad \alpha = 1, 2, 3, 4. \quad (3.15)$$

With these unit vectors we define a set of four three-preon states in isospinor-superspinor space by

$$\begin{aligned} \Theta_{\alpha\alpha'\alpha''}^1 &:= \frac{1}{\sqrt{3}} \left[\begin{pmatrix} 1 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 3 \\ \alpha' \end{pmatrix} \begin{pmatrix} 4 \\ \alpha'' \end{pmatrix} \right\}_s \right. \\ &\quad \left. + \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 4 \\ \alpha' \end{pmatrix} \begin{pmatrix} 1 \\ \alpha'' \end{pmatrix} \right\}_s + \begin{pmatrix} 4 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 1 \\ \alpha' \end{pmatrix} \begin{pmatrix} 3 \\ \alpha'' \end{pmatrix} \right\}_s \right], \end{aligned}$$

$$\begin{aligned} \Theta_{\alpha\alpha'\alpha''}^2 &:= \frac{1}{\sqrt{3}} \left[\begin{pmatrix} 2 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 3 \\ \alpha' \end{pmatrix} \begin{pmatrix} 4 \\ \alpha'' \end{pmatrix} \right\}_s \right. \\ &\quad \left. + \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 4 \\ \alpha' \end{pmatrix} \begin{pmatrix} 2 \\ \alpha'' \end{pmatrix} \right\}_s + \begin{pmatrix} 4 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ \alpha' \end{pmatrix} \begin{pmatrix} 3 \\ \alpha'' \end{pmatrix} \right\}_s \right], \end{aligned}$$

$$\begin{aligned} \Theta_{\alpha\alpha'\alpha''}^3 &:= \frac{1}{\sqrt{3}} \left[\begin{pmatrix} 1 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ \alpha' \end{pmatrix} \begin{pmatrix} 3 \\ \alpha'' \end{pmatrix} \right\}_s \right. \\ &\quad \left. + \begin{pmatrix} 2 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ \alpha' \end{pmatrix} \begin{pmatrix} 1 \\ \alpha'' \end{pmatrix} \right\}_s + \begin{pmatrix} 3 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 1 \\ \alpha' \end{pmatrix} \begin{pmatrix} 2 \\ \alpha'' \end{pmatrix} \right\}_s \right], \end{aligned}$$

$$\begin{aligned} \Theta_{\alpha\alpha'\alpha''}^4 &:= \frac{1}{\sqrt{3}} \left[\begin{pmatrix} 1 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ \alpha' \end{pmatrix} \begin{pmatrix} 4 \\ \alpha'' \end{pmatrix} \right\}_s \right. \\ &\quad \left. + \begin{pmatrix} 2 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 4 \\ \alpha' \end{pmatrix} \begin{pmatrix} 1 \\ \alpha'' \end{pmatrix} \right\}_s + \begin{pmatrix} 4 \\ \alpha \end{pmatrix} \left\{ \begin{pmatrix} 1 \\ \alpha' \end{pmatrix} \begin{pmatrix} 2 \\ \alpha'' \end{pmatrix} \right\}_s \right], \end{aligned} \quad (3.16)$$

and its “hypercharge” conjugated set by

$$\hat{\Theta}^h = \sum_j C^{hj} \Theta^j, \quad (3.17)$$

where C is the ordinary charge conjugation matrix in spinspace, i.e. we transfer this definition into the space of the four states (3.16).

Apart from the problem of left-handed and right-handed components of fermions which will be treated in the next section, by the definitions given above the electroweak boson as well as the lepton and quark states are completely fixed. Thus, we finally need the identification and interpretation of these states. For boson states we have

$$b \begin{pmatrix} 0 \\ \mathbf{k} \\ d \\ e \end{pmatrix} =: b_d^e(\mathbf{k}) \Rightarrow U^{rr'} e^{i\mathbf{k}(\mathbf{r}+\mathbf{r}')/2} \begin{pmatrix} 1s \\ \mathbf{r}-\mathbf{r}' \end{pmatrix} S_{\alpha\alpha'}^d T_{\alpha\alpha'}^e \quad (3.18)$$

and with (3.7) we obtain the preon content and the phenomenological particle interpretation

$$\begin{aligned} b_d^0(\mathbf{k}) &\Rightarrow \{\mathbf{P}\bar{\mathbf{P}}\}_{\text{as}} + \{\mathbf{N}\bar{\mathbf{N}}\}_{\text{as}} \Rightarrow W_d^0(\mathbf{k}), \\ b_d^1(\mathbf{k}) &\Rightarrow \{\mathbf{P}\bar{\mathbf{N}}\}_{\text{as}} + \{\mathbf{N}\bar{\mathbf{P}}\}_{\text{as}} \Rightarrow W_d^1(\mathbf{k}), \\ b_d^2(\mathbf{k}) &\Rightarrow i\{\bar{\mathbf{N}}\mathbf{P}\}_{\text{as}} + i\{\mathbf{N}\bar{\mathbf{P}}\}_{\text{as}} \Rightarrow W_d^2(\mathbf{k}), \\ b_d^3(\mathbf{k}) &\Rightarrow \{\mathbf{P}\bar{\mathbf{P}}\}_{\text{as}} - \{\mathbf{N}\bar{\mathbf{N}}\}_{\text{as}} \Rightarrow W_d^3(\mathbf{k}). \end{aligned} \quad (3.19)$$

The indices d refer to the spin-1 state representation (vector potential, field strength tensor) and were discussed in [25]. It is convenient to introduce

$$\begin{aligned} b_d^1(\mathbf{k}) + i b_d^2(\mathbf{k}) &\Rightarrow 2\{\mathbf{P}\bar{\mathbf{N}}\}_{\text{as}} \Rightarrow W_d^+(\mathbf{k}), \\ b_d^1(\mathbf{k}) - i b_d^2(\mathbf{k}) &\Rightarrow 2\{\mathbf{N}\bar{\mathbf{P}}\}_{\text{as}} \Rightarrow W_d^-(\mathbf{k}). \end{aligned} \quad (3.20)$$

The charges of these bosons are

$$Q = \begin{matrix} & 0 & 1 & -1 & 0 \\ W_d^0(\mathbf{k}) & W_d^+(\mathbf{k}) & W_d^-(\mathbf{k}) & W_d^3(\mathbf{k}) \end{matrix}. \quad (3.21)$$

The fermion states are given by the expressions

$$f \begin{pmatrix} q \\ l \\ j \end{pmatrix} =: {}^q f_l^j(\mathbf{q}) \Rightarrow U^{rr'r''} e^{i\mathbf{q}(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')^{1/3}} \Theta_{\alpha\alpha'\alpha''}^j \quad (3.22)$$

$$\cdot \left\{ \begin{pmatrix} l_1 \\ \alpha \end{pmatrix} \begin{pmatrix} l_2 \\ \alpha' \end{pmatrix} \begin{pmatrix} l_3 \\ \alpha'' \end{pmatrix} \begin{pmatrix} 1s \\ \mathbf{r}' - \mathbf{r}'' \end{pmatrix} \begin{pmatrix} 2p_q \\ \mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'') \end{pmatrix} \right\}_{\text{as}},$$

$$q = 0, 1$$

with $(2p_0) \equiv (2s)$ and

$${}^q f_l^h(\mathbf{q}) \Rightarrow U^{rr'r''} e^{i\mathbf{q}(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')^{1/3}} \hat{\Theta}_{\alpha\alpha'\alpha''}^h \quad (3.23)$$

$$\cdot \left\{ \begin{pmatrix} l_1 \\ \alpha \end{pmatrix} \begin{pmatrix} l_2 \\ \alpha' \end{pmatrix} \begin{pmatrix} l_3 \\ \alpha'' \end{pmatrix} \begin{pmatrix} 1s \\ \mathbf{r}' - \mathbf{r}'' \end{pmatrix} \begin{pmatrix} 2p_q \\ \mathbf{r} - \frac{1}{2}(\mathbf{r}' + \mathbf{r}'') \end{pmatrix} \right\}_{\text{as}},$$

$$q = 2, 3.$$

With (3.16) and (3.17) we then obtain the preon content and the phenomenological particle interpretation

	$q=0$	$q=1$	Charge Q
${}^q f_l^1(\mathbf{q}) \Rightarrow \{\bar{N}P\bar{P}\}_s \Rightarrow \nu$	u_1	0	
${}^q f_l^2(\mathbf{q}) \Rightarrow \{\bar{P}N\bar{N}\}_s \Rightarrow e$	d_1	-1	
${}^q f_l^3(\mathbf{q}) \Rightarrow \{N\bar{P}\bar{P}\}_s \Rightarrow \bar{\nu}$	\bar{u}_1	0	
${}^q f_l^4(\mathbf{q}) \Rightarrow \{P\bar{N}\bar{N}\}_s \Rightarrow \bar{e}$	\bar{d}_1	1	

$$(3.24)$$

and

	$q=2$	$q=3$	Charge Q
${}^q f_l^1(\mathbf{q}) \Rightarrow \{P\bar{N}\bar{N}\}_s \Rightarrow u_2$	u_3	1	
${}^q f_l^2(\mathbf{q}) \Rightarrow \{N\bar{P}\bar{P}\}_s \Rightarrow d_2$	d_3	0	
${}^q f_l^3(\mathbf{q}) \Rightarrow \{\bar{P}N\bar{N}\}_s \Rightarrow \bar{u}_2$	\bar{u}_3	-1	
${}^q f_l^4(\mathbf{q}) \Rightarrow \{\bar{N}P\bar{P}\}_s \Rightarrow \bar{d}_2$	\bar{d}_3	0	

$$(3.25)$$

The indices l refer to the spin 1/2-state representation and will be discussed in the next section. The index $q=1, 2, 3$ denotes the color degree of freedom. Due to the integer charge of P , the quark states have integer charges, too. Hence the quark states must be Han-Nambu quarks. For Han-Nambu quarks the charges and hypercharges are different between $q=0, 1$ and $q=2, 3$. Hypercharge and its different values can be derived directly from the effective coupling and need not be introduced especially. A discussion of integer charged quarks,

their different hypercharges etc. is given by Hendry and Lichtenberg [37], the original proposal is due to Han and Nambu [38].

Equations (3.24), (3.25) represent the first generation of leptons and quarks. Within the framework of a preon field theory a lot of proposals were published how to understand the existence of higher generations. As all these proposals are purely speculative we prefer to study the properties of the first generation more thoroughly before turning to the problem of higher generations.

4. Effective Fermion-Boson Coupling

The effective coupling of fermions and bosons can be derived by a further evaluation of the coupling terms (2.7), if the explicit expressions for the fermion and boson states (3.18), (3.22) and (3.23) are substituted into (2.7). For a first step in this paper we restrict ourselves to the discussion of left-handed fermion states. As will be shown later, these states follow from the v -expansion of (3.11). Furthermore, we first discuss the first term of (2.7). Observing the definitions (3.18) and (3.22) and denoting the first term of (2.7) by $(\mathcal{H}_{bf}^1)_1$ we need for the further evaluation the full indexing and can write this term in the form

$$(\mathcal{H}_{bf}^1)_1 = 3 \sum_h \int \hat{W}^h \begin{pmatrix} \mathbf{r}, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4 \\ r, r_2, r_3, r_4 \\ \alpha, \alpha_2, \alpha_3, \alpha_4 \\ \chi, \chi_2, \chi_3, \chi_4 \end{pmatrix}$$

$$\cdot C \begin{pmatrix} \mathbf{r}_2, \mathbf{r}_3 \\ r_2, r_3 \\ \alpha_2, \alpha_3 \\ \chi_2, \chi_3 \end{pmatrix} \begin{pmatrix} 0 \\ \mathbf{p} \\ d \\ e \end{pmatrix} C \begin{pmatrix} \mathbf{r}_4, s_1, s_2 \\ r_4, s_1, s_2 \\ \alpha_4, v_1, v_2 \\ \chi_4, \mu_1, \mu_2 \end{pmatrix} \begin{pmatrix} q \\ l \\ j \end{pmatrix}$$

$$\cdot R \begin{pmatrix} \mathbf{r}, s_1, s_2 \\ r, s_1, s_2 \\ \alpha, v_1, v_2 \\ \chi, \mu_1, \mu_2 \end{pmatrix} \begin{pmatrix} q' \\ \mathbf{q}' \\ l' \\ j' \end{pmatrix} {}^q f_l^j(\mathbf{q}) \frac{\delta}{\delta {}^q f_l^j(\mathbf{q})} \frac{\delta}{\delta b_d^q(\mathbf{p})}$$

$$\cdot dp dq dq' dr dr_2 dr_3 dr_4 ds_1 ds_2, \quad (4.1)$$

where for all doubly occurring indices the summation convention is used. As the wave functions (3.18), (3.22) and (3.23) are orthonormal the dual set of wave functions $\{R_s\}$ is identical with the original set $\{C_s\}$. Hence by substitution of these wave functions and of the vertex expression (1.16) into (4.1)

this expression can be evaluated straightforwardly. The substitution of these expressions yields the following formula for (4.1)

$$\begin{aligned}
 (\mathcal{H}_{bf}^1)_1 &= 3 \sum (-1)^r U^{r_2 r_3} U^{r_4 s_1 s_2} S^{r s_1 s_2} \\
 &\cdot \sum_h \hat{V}^h(\alpha, \alpha_2, \alpha_3, \alpha_4) T_{\alpha_2 \alpha_3}^e \Theta_{\alpha_4 \mu_1 \mu_2}^j \Theta_{\alpha \mu_1 \mu_2}^{j'} \\
 &\cdot \int \delta(\mathbf{r} - \mathbf{r}_2) \delta(\mathbf{r} - \mathbf{r}_3) \delta(\mathbf{r} - \mathbf{r}_4) S_{\alpha_2 \alpha_3}^d e^{i\mathbf{p}(\mathbf{r}_2 + \mathbf{r}_3) \cdot \mathbf{r}} \left(\frac{1s}{\mathbf{r}_2 - \mathbf{r}_3} \right) \\
 &\cdot \left\{ \chi_{\alpha_4 \nu_1 \nu_2}^{l_1 l_2 l_3} e^{i\mathbf{q}(\mathbf{r}_4 + \mathbf{s}_1 + \mathbf{s}_2) \cdot \mathbf{r}} \left(\frac{1s}{\mathbf{s}_1 - \mathbf{s}_2} \right) \left(\frac{2\mathbf{p}_e}{\mathbf{r}_4 - \frac{1}{2}(\mathbf{s}_1 + \mathbf{s}_2)} \right) \right\}_{\text{as}} \\
 &\cdot \left\{ \chi_{\alpha \nu_1 \nu_2}^{l_1 l_2 l_3} e^{-i\mathbf{q}'(\mathbf{r} + \mathbf{s}_1 + \mathbf{s}_2) \cdot \mathbf{r}} \left(\frac{1s}{\mathbf{s}_1 - \mathbf{s}_2} \right) \left(\frac{2\mathbf{p}_{e'}}{\mathbf{r} - \frac{1}{2}(\mathbf{s}_1 + \mathbf{s}_2)} \right) \right\}_{\text{as}} \\
 &\cdot \Theta_{f_r}^{j'}(\mathbf{q}') \frac{\delta}{\delta \Theta_{f_l}^j(\mathbf{q})} \frac{\delta}{\delta b_\mu^e(\mathbf{p})} \\
 &\cdot d\mathbf{p} d\mathbf{q} d\mathbf{q}' d\mathbf{r} d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{s}_1 d\mathbf{s}_2 \quad (4.2)
 \end{aligned}$$

with $\Theta^j \equiv \Theta^j$ for $j = 0, 1$ and $\Theta^j \equiv \hat{\Theta}^j$ for $j = 2, 3$ and

$$\begin{aligned}
 \sum_h \hat{V}^h(\alpha, \alpha_2, \alpha_3, \alpha_4) \quad (4.3) \\
 = \{ -\delta_{\alpha \alpha_2} \gamma_{\alpha_3 \alpha_4}^5 [\gamma_{\alpha \alpha_2}^0 C_{\alpha_3 \alpha_4} - (\gamma^0 \gamma^5)_{\alpha \alpha_2} (\gamma^5 C)_{\alpha_3 \alpha_4}] \\
 + \delta_{\alpha \alpha_3} \gamma_{\alpha_2 \alpha_4}^5 [\gamma_{\alpha \alpha_3}^0 C_{\alpha_2 \alpha_4} - (\gamma^0 \gamma^5)_{\alpha \alpha_3} (\gamma^5 C)_{\alpha_2 \alpha_4}] \\
 - \delta_{\alpha \alpha_4} \gamma_{\alpha_2 \alpha_3}^5 [\gamma_{\alpha \alpha_4}^0 C_{\alpha_2 \alpha_3} - (\gamma^0 \gamma^5)_{\alpha \alpha_4} (\gamma^5 C)_{\alpha_2 \alpha_3}] \}.
 \end{aligned}$$

The calculation of the sum over the auxiliary field factors was performed in [24]. One obtains

$$\sum_{\substack{r_2 r_3 r_4 \\ s_1 s_2}} (-1)^r U^{r_2 r_3} U^{r_4 s_1 s_2} U^{r s_1 s_2} = g_1 \eta^{-2} \quad (4.4)$$

with $\eta := m/\Delta m$ and g_1 as a numerical factor. Furthermore, due to $\text{tr}(\gamma^5 T^e) = 0$ in the vertex part (4.3) the last term contributes nothing while the second term can be shown to be identical with the first term. Hence we can study the expression

$$\begin{aligned}
 W_1 \left(\frac{e, j, j'}{d, \alpha_4, \alpha} \right)_{\Theta^j} \\
 := -2 \delta_{\alpha \alpha_2} \gamma_{\alpha_3 \alpha_4}^5 [\gamma_{\alpha \alpha_2}^0 C_{\alpha_3 \alpha_4} - (\gamma^0 \gamma^5)_{\alpha \alpha_2} (\gamma^5 C)_{\alpha_3 \alpha_4}] \\
 \cdot S_{\alpha_2 \alpha_3}^d T_{\alpha_2 \alpha_3}^e \Theta_{\alpha_4 \mu_1 \mu_2}^j \Theta_{\alpha \mu_1 \mu_2}^{j'}. \quad (4.5)
 \end{aligned}$$

The spin states of the electroweak vector bosons are given by the set (3.6). Without reference to a special representation it follows by substitution of (3.6) into (4.5) that only the vector part of (3.6) leads to a non-vanishing contribution, while the tensor part cancels completely in (4.5), i.e. we have a coupling of the vector potential to the fermions while the

field strengths do not couple to fermions. Thus we obtain from (4.5)

$$W_1 \left(\frac{e, j, j'}{\mu, \alpha_4, \alpha} \right)_{\Theta^j} = W_1(e, j, j')_{\Theta^j} (\gamma^0 \gamma^\mu)_{\alpha \alpha_4} \quad (4.6)$$

for $\{d \equiv \mu\}$ and zero for $\{d \equiv \mu, \nu\}$ with

$$W_1(e, j, j')_{\Theta^j} := 4 \delta_{\alpha \alpha_2} \gamma_{\alpha_3 \alpha_4}^5 T_{\alpha_2 \alpha_3}^e \Theta_{\alpha_4 \mu_1 \mu_2}^j \Theta_{\alpha \mu_1 \mu_2}^{j'} \quad (4.7)$$

and after integration over $\mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4$, (4.2) goes over into

$$\begin{aligned}
 (\mathcal{H}_{bf}^1)_1 &= 6 g_1 \eta^{-2} m^{3/2} W_1(e, j, j')_{\Theta^j} (\gamma^0 \gamma^\mu)_{\alpha \alpha_4} \\
 &\cdot \int d\mathbf{p} d\mathbf{q} d\mathbf{q}' d\mathbf{r} d\mathbf{s}_1 d\mathbf{s}_2 \\
 &\cdot e^{i\mathbf{p} \cdot \mathbf{r}} \left\{ \chi_{\alpha_4 \nu_1 \nu_2}^{l_1 l_2 l_3} e^{i\mathbf{q}(\mathbf{r} + \mathbf{s}_1 + \mathbf{s}_2) \cdot \mathbf{r}} \left(\frac{1s}{\mathbf{s}_1 - \mathbf{s}_2} \right) \left(\frac{2\mathbf{p}_e}{\mathbf{r} - \frac{1}{2}(\mathbf{s}_1 + \mathbf{s}_2)} \right) \right\}_{\text{as}} \\
 &\cdot \left\{ \chi_{\alpha \nu_1 \nu_2}^{l_1 l_2 l_3} e^{-i\mathbf{q}'(\mathbf{r} + \mathbf{s}_1 + \mathbf{s}_2) \cdot \mathbf{r}} \left(\frac{1s}{\mathbf{s}_1 - \mathbf{s}_2} \right) \left(\frac{2\mathbf{p}_{e'}}{\mathbf{r} - \frac{1}{2}(\mathbf{s}_1 + \mathbf{s}_2)} \right) \right\}_{\text{as}} \\
 &\cdot \Theta_{f_r}^{j'}(\mathbf{q}') \frac{\delta}{\delta \Theta_{f_l}^j(\mathbf{q})} \frac{\delta}{\delta b_\mu^e(\mathbf{p})}. \quad (4.8)
 \end{aligned}$$

We now assume the spin 1/2 fermions to be generated by coupling a spin zero preon singlet with $l_2 = 1/2, l_3 = -1/2$ to a spin 1/2 preon with spin direction l_1 (always for v -spinors!). Then the anti-symmetrized fermion parts in (3.10) or (4.8) respectively are well defined and their integrals can be evaluated along the lines which were outlined in [24]. For short we do explicitly not repeat this procedure for the case under consideration but give only the result. We obtain from (4.8) the expression

$$\begin{aligned}
 (\mathcal{H}_{bf}^1)_1 &= 6 g_1 \eta^{-2} m^{3/2} W_1(e, j, j')_{\Theta^j} \left(\frac{l_1}{\alpha} \right) (\gamma^0 \gamma^\mu)_{\alpha \alpha_4} \left(\frac{l_1}{\alpha_4} \right) \\
 &\cdot \int e^{i\mathbf{z} \cdot (\mathbf{p} + \mathbf{q} - \mathbf{q}')} \Theta_{f_l}^{j'}(\mathbf{q}') \frac{\delta}{\delta \Theta_{f_l}^j(\mathbf{q})} \frac{\delta}{\delta b_\mu^e(\mathbf{p})} \\
 &\cdot d\mathbf{z} d\mathbf{p} d\mathbf{q} d\mathbf{q}'. \quad (4.9)
 \end{aligned}$$

For the further evaluation of (4.9) we remember that in a first step we will only discuss v -solutions. Thus we have $(\frac{l_1}{\alpha}) \equiv v_\alpha(l)$ and according to Sect. 3 in the low energy limit these spinors are given by $v_\alpha(\frac{1}{2}) := \delta_{\alpha 3}$ and $v_\alpha(-\frac{1}{2}) := \delta_{\alpha 4}$. On the other hand, all calculations which are concerned with the spinor algebra itself were done representation-free. Hence we can choose any representation which is suitable for our purposes. We consider the chiral representation. If the Dirac spinors in this representation are denoted by $v_\alpha(l)_{\text{Ch}}$, then in the low energy limit we

have

$$(\mathbf{1} - \gamma^5)_{\text{Ch}} \cdot v(l)_{\text{Ch}} \equiv v(l) \equiv [v(l)_{\text{Ch}}]_{\text{L}} \quad (4.10)$$

i.e. our expansion states $v_x(l)$ are identical with the left-handed components of the chiral representation. Thus for this representation we have the identity

$$(l'_x) (\gamma^0 \gamma^\mu)_{xx'}^{\text{Ch}} (l'_x) \equiv [v(l'_1)_{\text{Ch}}]_{\text{L}} \cdot (\gamma^0 \gamma^\mu)_{\text{Ch}} \cdot [v(l_1)_{\text{Ch}}]_{\text{L}}. \quad (4.11)$$

In (4.11) the right-hand side is invariant against the change of the representation. Hence

$$(l'_x) (\gamma^0 \gamma^\mu)_{xx'}^{\text{Ch}} (l'_x) = [v(l'_1)]_{\text{L}} \cdot (\gamma^0 \gamma^\mu) \cdot [v(l_1)]_{\text{L}} \quad (4.12)$$

holds for any representation, and starting with a chiral representation for the γ -algebra we can substitute (4.12) into (4.9). If then the Fourier-transforms

$$e f_x^j(\mathbf{r})_{\text{L}} = \sum_l \int e^{-i \mathbf{q} \cdot \mathbf{r}} [v_x(l)]_{\text{L}} e f_l^j(\mathbf{q}) d\mathbf{q} \quad (4.13)$$

etc. are introduced we can rewrite (4.9) into the form

$$(\mathcal{H}_{bf}^1)_{\text{L}} = \tilde{g}_1 W_1(e, j, j')_{ee'} \cdot \int e f_x^{j'}(\mathbf{r})_{\text{L}} (\gamma^0 \gamma^\mu)_{xx'} \frac{\delta}{\delta e f_x^j(\mathbf{r})_{\text{L}}} \frac{\delta}{\delta b_\mu^e(\mathbf{r})} d\mathbf{r} \quad (4.14)$$

with $\tilde{g}_1 = 6 g_1 \eta^{-2} m^{3/2}$.

Finally, we have to evaluate (4.7). A straightforward calculation for the various elements of the sets (3.7) and (3.16) or (3.17) respectively yields the following expression for $q = 0, 1$

$$(\mathcal{H}_{bf}^1)_{\text{L}}^A = \sum_{q=0,1} \tilde{g}_1 \int e f_x^u(\mathbf{r})_{\text{L}} (\gamma^0 \gamma^\mu)_{xx'} \cdot \left[g' Y_q \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{uu'} \frac{\delta}{\delta b_\mu^0(\mathbf{r})} + g \sum_t \begin{pmatrix} \sigma_t & 0 \\ 0 & (-1) \sigma_t \end{pmatrix}_{uu'} \frac{\delta}{\delta b_\mu^t(\mathbf{r})} \right] \frac{\delta}{\delta e f_x^{u'}(\mathbf{r})_{\text{L}}} d\mathbf{r} \quad (4.15)$$

with $g' = g = 1$ and $Y_q = -1$ for $q = 0, 1$.

For $q = 2, 3$, on the other hand, we obtain the expression

$$(\mathcal{H}_{bf}^1)_{\text{L}}^B = \sum_{q=2,3} \tilde{g}_1 \int e f_x^u(\mathbf{r})_{\text{L}} (\gamma^0 \gamma^\mu)_{xx'} \cdot \left[g' Y_q \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{uu'} \frac{\delta}{\delta b_\mu^0(\mathbf{r})} + g \sum_t \begin{pmatrix} \sigma_t & 0 \\ 0 & (-1) \sigma_t \end{pmatrix}_{uu'} \frac{\delta}{\delta b_\mu^t(\mathbf{r})} \right] \frac{\delta}{\delta e f_x^{u'}(\mathbf{r})_{\text{L}}} d\mathbf{r} \quad (4.16)$$

with $g' = g = 1$, but $Y_q = 1$ for $q = 2, 3$.

Then the total effective interaction is given by

$$(\mathcal{H}_{bf}^1)_{\text{L}} = (\mathcal{H}_{bf}^1)_{\text{L}}^A + (\mathcal{H}_{bf}^1)_{\text{L}}^B. \quad (4.17)$$

The same procedure can be applied to the second term of (2.7) and by a straightforward calculation it can be shown that in the leading term approximation this term can nothing contribute to (2.7). Hence we have

$$(\mathcal{H}_{bf}^1) \equiv (\mathcal{H}_{bf}^1)_{\text{L}} \quad (4.18)$$

in this approximation. For short we do not explicitly demonstrate this in this paper. Thus we have only to compare (4.15) and (4.16) with the phenomenological expression.

For a comparison we start with the original Weinberg-Salam Lagrangian for left-handed lepton and quark spinors before any symmetry breaking and the Weinberg gauge-boson combination is introduced. Denoting the doublets by

$$\psi_{\text{L}} := \begin{pmatrix} \nu_{\text{L}} \\ e_{\text{L}} \end{pmatrix}; \quad \chi_{\text{L}}^q := \begin{pmatrix} u_{\text{L}}^q \\ d_{\text{L}}^q \end{pmatrix}; \quad q = 1, 2, 3 \quad (4.19)$$

and introducing superspinors by

$$\Psi_{\text{L}} := \begin{pmatrix} \psi_{\text{L}} \\ \psi_{\text{L}}^c \end{pmatrix}; \quad \mathbf{X}_{\text{L}}^q := \begin{pmatrix} \chi_{\text{L}}^q \\ \chi_{\text{L}}^{qc} \end{pmatrix} \quad (4.20)$$

we obtain from the Weinberg-Salam Lagrangian for leptons and Han-Nambu quarks the following Dirac equations

$$i \partial_0 \begin{pmatrix} \Psi \\ \mathbf{X}^1 \end{pmatrix}_{\text{L}} = \left[-i \nabla + g \sum_t \begin{pmatrix} \sigma_t & 0 \\ 0 & (-1)^t \sigma_t \end{pmatrix} \mathbf{W}^t - g' \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \mathbf{W}_0 \right] \cdot \gamma^0 \bar{\gamma} \begin{pmatrix} \Psi \\ \mathbf{X}^1 \end{pmatrix}_{\text{L}} \quad (4.21)$$

and

$$i \partial_0 \begin{pmatrix} \mathbf{X}^2 \\ \mathbf{X}^3 \end{pmatrix}_{\text{L}} = \left[-i \nabla + g \sum_t \begin{pmatrix} \sigma_t & 0 \\ 0 & (-1)^t \sigma_t \end{pmatrix} \mathbf{W}^t - g^1 \begin{pmatrix} -\sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \mathbf{W}_0 \right] \cdot \gamma^0 \bar{\gamma} \begin{pmatrix} \mathbf{X}^2 \\ \mathbf{X}^3 \end{pmatrix}_{\text{L}}. \quad (4.22)$$

Formulating these equations in the functional energy representation one obtains for the coupling terms the expressions (4.15) and (4.16) if a temporal gauge is used, i.e. the shell model effective electroweak coupling terms coincide with the phenomenological electroweak coupling terms of fermions and bosons.

With respect to (4.21) and (4.22) it has, however, to be emphasized that these equations arise by the phenomenological introduction and definition of hypercharge, cf. for instance Cheng and Li [39], whereas in the shell model coupling terms (4.15) and (4.16) hypercharge is generated by the formalism, i.e. need not be introduced additionally. The same holds true for color: Also these degrees of freedom have phenomenologically to be introduced by definition which leads to (4.21) and (4.22),

whereas in the coupling terms (4.15) and (4.16) color is generated by the shell model states themselves.

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